MATH 590: QUIZ 7 SOLUTIONS

Name:

1. For an $n \times n$ matrix A with entries in $\mathbb{C}$, define: (i) The characteristic polynomial $p_A(x)$; (ii) The eigenvalues of A; (iii) The eigenspace E_λ, for λ an eigenvalue of A. (3 points)

Solution. (i) $p_A(x) = |A - xI_n|$ or $|xI_n - A|$.

(ii) $\lambda \in \mathbb{C}$ is an eigenvalue of A if $p_A(\lambda) = 0$.

(iii) For the eigenvalue λ, $E_\lambda := \{v \in \mathbb{C}^n \mid Av = \lambda v\}$.

2. For the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + 2y, 2x + y)$, verify that T is a symmetric linear transformation by applying the definition of symmetric linear transformation to the vectors $v_1 := (a, b)$ and $v_2 := (c, d)$. (7 points)

Solution. One must show that $T(v_1) \cdot v_2 = v_1 \cdot T(v_2)$.

$T(v_1) \cdot v_2 = (a + 2b, 2a + b) \cdot (c, d) = ac + 2bc + 2ad + bd$.

$v_1 \cdot T(v_2) = (a, b) \cdot (c + 2d, 2c + d) = ac + 2ad + 2bc + bd$.

Thus, $T(v_1) \cdot v_2 = v_1 \cdot T(v_2)$, showing that T is a symmetric linear transformation.